doi 10.18699/bgrs2024-2.3-06

Mean-field control models for describing information dissemination in online social networks

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Key words: inverse problem, optimization, mean-field games, social process

Motivation and Aim: Mathematical models of the dynamics of social processes are poorly understood and rely on systems of differential equations [1, 2] and agent-based models [3]. Due to incomplete data, the use of these models does not fully characterize the process and its control. It is necessary to use the mean-field methodology [4] based on the dual game that allows one optimal control the information dissemination in online social networks. We consider a large number of users who can take states $x \in [0,1]$, where 0 means that the user is involved in the information dissemination process and 1 means the opposite. Then the density of users $u(x,t)\colon [0,1] \times [1,T] \to \mathbb{R}$ satisfies the Kolmogorov (Fokker-Planck) equation

$$u_t + \nabla(u\alpha) + \left(\frac{u}{K_{cap}} - 1\right) r(t)u - Du_{xx} = 0,\ x \in [0,1], \quad t \in [1,T] \qquad (1)$$

with initial condition and Robin boundary conditions

$$u(x,1) = \varphi(x),\ \ x \in [0,1] \qquad (2)$$

$$\beta_1\, u_x\,(0,t) - \beta_2\, u\,(0,t) = 0,\ \ u_x\,(1,t) = 0,\ \ t \in [1,T]. \qquad (3)$$

Here, $\alpha(x,t)\colon [0,1] \times [1,T] \to \mathbb{R}$ is the control parameter (in other words, the user strategy) that ensures the Nash equilibrium of the system of interacting agents and minimizes the cost functional with respect to (u,α)

$$J(u,\alpha) = \int_1^T \int_0^1 \left(d_1 e^{-t} \frac{u\alpha^2}{2} + d_2 (x-1)^2 u \right) dxdt.$$

Here d_1 and d_2 are the weight coefficients, K_{cap} is a carrying capacity, which is the maximum possible density of influenced users at a given distance, $r(t)$ is a growth rate of the number of influenced users placed at the same distance away from the source, D is a popularity of information which promotes the spread of the information through non-structure based activities such as search.

For $\alpha = 0$, equation (1) is investigated in [5].

Methods and Algorithms: Using the Lagrange multiplier method [6], a system similar to the Hamilton-Jacobi-Bellman equation is constructed:

$$\begin{gathered} v_t + \alpha\, v_x + \left(1 - \frac{2u}{K_{cap}}\right) r(t)v + Dv_{xx} = -d_1 e^{-t} \frac{\alpha^2}{2} - d_2 (x-1)^2, \\ v(x,T) = 0,\ \ x \in [0,1], \\ v_x\,(0,t) = v_x\,(1,t) = 0, \quad t \in [1,T]. \end{gathered} \qquad (4)$$

The optimality conditions are defined from following equations

$$d_1 e^{-t}\alpha + v_x = 0, \quad \alpha(0,t) = \alpha(1,t) = 0. \quad (5)$$

Problem (1)–(3), (4), (5) were solved by the finite-difference scheme proposed in [7].
To solve the inverse problem, it is necessary to determine the functions (φ, α) by additional measurements $u(x,t)$ in the computational domain. The identifiability of the model was analyzed by the Sobol sensitivity analysis method.
For the case $\alpha = 0$ (without control) we minimized the functional

$$J(\varphi) = \frac{(T-1)}{N_2} \sum_{k=1}^{N_2} \left| \sum_{i=1}^{N_1} u(x_i, t_k; \varphi) - f_k \right|^2 \quad (6)$$

using:

- the tensor optimization method (TT),
- the combination of TT and fast gradient method (FGM),
- the combined method with A.N. Tikhonov regularization,
- the combination of stochastic particle swarm optimization (PSO) and FGM.

Results: The following results were obtained for the case $\alpha = 0$ (without control). Figure 1 shows the results of recovering the initial density function $\varphi(x)$ at 6 points using: the tensor optimization method (TT) – red line with triangles, the combination of TT and fast gradient method (FGM) – green line with squares, the combined method with A.N. Tikhonov regularization – blue line with pentagons, and the combination of stochastic particle swarm optimization (PSO) and FGM – orange line with rhombuses.

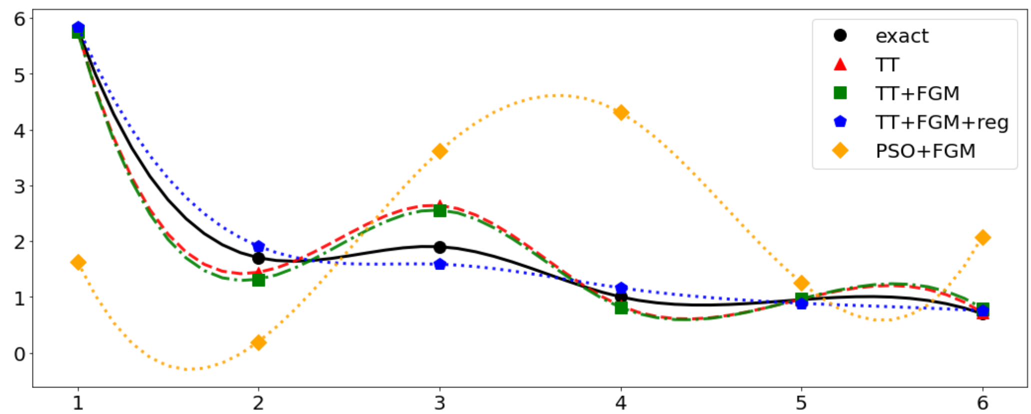
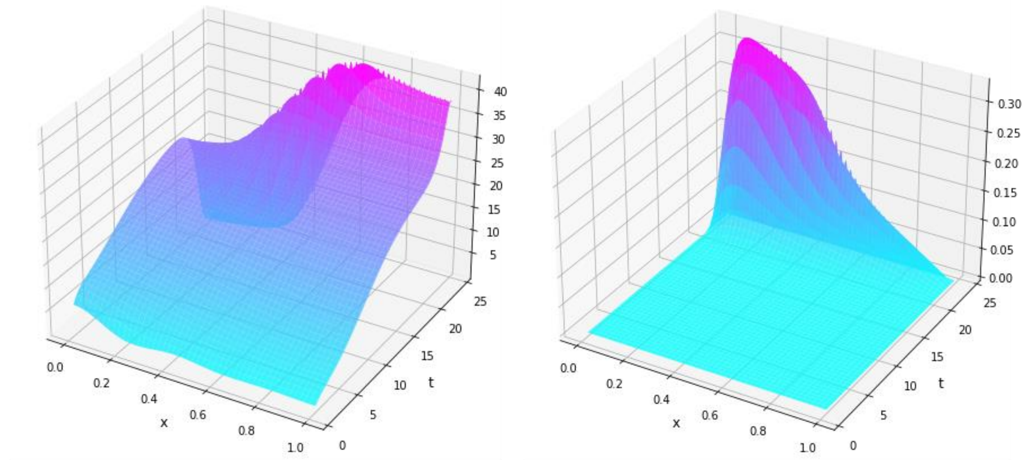


Fig. 1. Results of reconstruction of the initial density function $\varphi(x)$ for the case $\alpha = 0$ (without control)

The results of the numerical solution of the direct problem (determining the functions $u(x,t)$ and $\alpha(x,t)$ of problem (1)-(3), (4), (5) at a given $\varphi(x)$) were obtained. Figure 2 shows graphs (a) of the density of involved users $u(x,t)$ and (b) of the control function $\alpha(x,t)$.



(a) (b)

Fig. 2. Plots of (a) the density of involved users $u(x,t)$ and (b) the control function $\alpha(x,t)$

Conclusion: For the case $\alpha = 0$ (without control) the best solution of the inverse problem in terms of error value is the solution obtained by the regularized combination of TT and FGM [8].

The aim of the control process was to minimize the number of users involved in the process of information dissemination on the left boundary. It is shown that at the found control α the goal is achieved (the number of users involved in the process of information dissemination at x close to 0 is reduced) at the terminal moment of time.

Funding: The study is supported by the Russian Science Foundation (project No. 23-71-10068).

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