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Optimal control models for solving problems of mathematical epidemiology

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Motivation and Aim: The Covid-19 pandemic has shown the need to develop strategies for regulating the policy by the planner (which, first of all, is government at various levels) of various social and economic measures to counter the virus. The practice of government decisions to curb the growth of morbidity, adopted in different countries, has shown that different strategies have different effects on the social and epidemiological situation (an example of various decisions and their consequences is discussed in [1–3]). Numerous examples are described in the social and epidemiological literature problems that arise when a social planner chooses a certain preventive strategy [4–6]. These include both social problems associated with distrust of information transmitted by health authorities and citizens' underestimation of the degree of risk, as well as economic restrictions associated with insufficient resources for carrying out quarantine measures. Thus, the problem of assessing scenarios for possible solutions for carrying out various antiviral measures in conditions of limited resources is relevant. So, the aim of this work is to show the result of many years of experience in using optimal control models in the field of mathematical epidemiology to solve problems of forecasting the spread of the virus, taking into account assumptions about the socio-economic characteristics of the region.

Methods and Algorithms: From the researcher's point of view, the solution to the above problems lies in the search for effective mathematical models that describe the situation, as well as methods for solving them. It is natural to assume that when solving problems of choosing optimal strategies, priority is given to mathematical models of optimal control. The report will consider two approaches to modeling the spread of epidemics developed by the authors: optimal control models with agent dynamics specified in the form of compartmental [7] epidemiological models, and the "mean field" models.

Mathematical "mean field" models, the original formulations of which were proposed in [8, 9], are becoming an increasingly popular tool for mathematical modeling. The advantage of the approach is the description of the collective behavior of a mass of agents (players) interacting in strategic situations using a small number of partial differential equations, which significantly reduces the calculation time and computational complexity of the problem compared to agent-based models when it comes to a large number of players. Mathematical epidemiology is no exception in this sense, since the task here is to describe the spread of a virus within a population represented by a large number of interacting individuals making strategic decisions (for example, about vaccination or compliance with specific quarantine restrictions). A broad overview of current work on the use of mean field models to simulate coronavirus dynamics is presented in [10].

In turn, optimal control models that depend only on time are not used so often to simulate the dynamics of the spread of an epidemic. This is apparently due to the fact that they differ structurally from compartmental epidemiological models only in the introduction of a control functional and a more complex set of numerical research methods. However, the practice of using such models shows a significant difference in forecasts depending on the type of model used. At the same time, simple optimal control models are more flexible in describing the system being modeled relative to the same "mean field" models, since the limitations imposed on them to justify the existence and uniqueness of the problem are less restrictive.

Results: The report will propose several formulations of the "mean field" problems and optimal control for the spread of the epidemic using the example of the Covid-19 pandemic in Novosibirsk and Krasnoyarsk with various assumptions about the social behavior of agents taking into account quarantine restrictions and the economic costs associated with the implementation of these restrictions. Their analytical and numerical comparison will be proposed.

Conclusion: Numerical comparison of optimal control models built on the basis of different approaches shows that "mean field" epidemiological models often give more accurate forecasts of the spread of morbidity compared to statistical data. Note, however, that "mean field" models are more sensitive to the choice of the problem functional compared to conventional optimal control models, since the domain of existence of the solution is significantly limited by the class of smooth Lipschitz functions, and the uniqueness of the solution can be guaranteed only in the case of strictly convex functionals and over short simulation periods. These limitations significantly complicate the selection of functional dependencies that describe the actual behavior of the economic-epidemiological system.

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