

Changes in the dynamics of a sex-structured population with non-overlapping generations during evolution

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Motivation and Aim: The present work is devoted to the development of an eco-genetic approach to modeling the dynamics of limited sex-structured populations with non-overlapping generations within mathematical biology. We propose a dynamic model of a population that is structured by sex, which allows for studying evolutionary processes [1]. Note that adding sex structure to the model is important for understanding population abundance fluctuations, since sexual reproduction is one of the essential mechanisms of microevolutionary transformations and animal adaptation to the environment. This paper aims to study the evolutionary dynamics of an ecologically limited population that is structured by sex with seasonal reproduction and non-overlapping generations. The reason for choosing populations with non-overlapping generations and a short life cycle with a limited breeding period is the occurrence of significant fluctuations in the abundance of such species.

Methods and Algorithms: This paper proposes and studies a discrete-time model for a sex-structured population with non-overlapping generations. The model assumes that density-dependent factors limit the survival of females and males at different intensities. In addition, we consider a population with genetic diversity in individuals' reproductive potentials, controlled by one autosomal diallelic locus. We consider a panmictic population with Mendelian inheritance rules. To study the model we used the analytical and numerical methods of dynamic systems studies.

Results: This study showed that an increase in the average value of the reproductive potential in a population with density-dependent regulation of survival destabilizes the dynamics of abundance of females and males. In this case, the scenario of stability loss of equilibrium via the period doubling or Neimark–Sacker bifurcations depends on the intensity of selfregulation. The genetic composition of the population, namely, monomorphic genotype fixation or coexistence of all genotypes in polymorphism, is shown to be determined by the values of the reproductive potentials of heterozygotes and homozygotes, initial conditions, and a parameter describing the ratio of newborn females to males. In particular, we found that the increased reproductive potential of heterozygotes leads to a stable polymorphism within a population. The intermediate value of the reproductive potential of heterozygotes results in a monomorphism with an allele with greater fitness. The reduced reproductive potential of heterozygotes induces bistable dynamics when the different monomorphic fixed points with zero and non-zero population sizes coexist. Thus, depending on the values of the initial allele frequencies and the parameter describing the ratio of newborn females to males, the population either becomes extinct or survives by fixing one of the alleles during evolution. In this scenario, the

polymorphic state is unstable and is part of a transitional process in the dynamics to achieve one of the monomorphic states. With the coexistence of two monomorphic equilibria with zero and non-zero population sizes, the model of a population with two sexes proposed in this study reveals the possibility of bottleneck effects and evolutionary rescue. This model reveals two variants of evolutionary rescue: when the population recovers due to rare alleles with higher fitness or the emergence of new alleles caused by mutations.

Conclusion: The growth rate at which the population survives and develops is shown to depend on the fitness of the genotypes and the secondary sex ratio. As a result, the asymptotic genetic composition of the population is determined by the values of the reproductive potentials of the heterozygote and homozygotes, the initial conditions, and the parameter describing the ratio of newborn females to males. With disruptive selection, the influence of external factors changing the current genetic composition of a population can alter the direction of evolution and lead to the extinction of a successful developing population or a gradual population recovery due to evolutionary rescue after a noticeable decline in its abundance [1].

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References

1. Revutskaya O., Neverova G., Zhdanova O., Frisman E. The Evolutionary Dynamics of a Sex-Structured Population with Non-Overlapping Generations. *Mathematics*. 2023;11(24):4971. doi 10.3390/math11244971