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# Fighting ticks with functional-analytic guns

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*Motivation and Aim*: Ticks and tick-borne diseases present a well-known threat to the health of people in many parts of the globe. The scientific literature devoted both to field observations and modelling the propagation of ticks continues to grow. So far, the majority of the mathematical studies were devoted to models based on ordinary differential equations, where spatial variability was taken into account by a discrete parameter. Only few papers use spatially nontrivial diffusion models, and they are devoted mostly to spatially homogeneous equilibria. Here we aim to develop diffusion models for the propagation of ticks stressing spatial heterogeneity. This allows us to assess the sizes of control zones that can be created (using various available techniques) to produce a patchy territory, on which ticks will be eventually eradicated.
*Methods and Algorithms*: The starting point for our analysis is the so-called KISS model for the description of several interacting species (developed for the analysis of planktons). However, we are interested in creating control zones (kind of barriers) with non-beneficial conditions that can stop the propagation of ticks. To this end, we first extend the KISS model to the case when the non-beneficial region is bounded, interpreting its size as an additional control parameter, and when the diffusion coefficient is discontinuous at the interface of beneficial and non-beneficial regions, together with the mortality coefficient. Then we consider the modifications of the model with various boundary conditions, most importantly with the periodic boundary conditions, which provide the key intermediate step to tackle our main questions: how to organise the sizes and properties of the regular patches of (intermittent) control zones that would prevent the growth of the population of ticks?
We suppose that in the control zone one is able to achieve the required level of the death rates or to decrease the reproduction rates of ticks. In literature one can find various methods that can be used locally for this purpose. It can be achieved by the treatment of the soil (planned burns, cleaning and decreasing humidity levels, chemical treatment), by tick-targeted strategies such as TickBots, by specific actions on hosts, like dipping of cattle with acaricide. Alternatively it can be achieved by biological methods like infesting woods with ants (related experiments on ants were conducted in USA and Russia), or the natural enemies of ticks like the insects Ixodiphagus hookeri.
From the mathematical point of view, what we are doing is getting criteria for negativity of a class of diffusion operators with piecewise continuous symbols. The negativity of the diffusion operators that govern the propagation of ticks ensures that the population will eventually die out (in the region considered).

*Results*: We give rather precise results for the required lengths of critical zones for a homogeneous population, and then extend them partially to the structured populations, with several stages, like larvae, nymphs and adults in ticks. Using averaged parameters taken from various field observations, we apply our theoretical results to the concrete cases of the lone star ticks of North America and of the taiga ticks of Russia, and thus give concrete assessment of barrier zones with concrete numbers.
*Conclusion*: The main goal was to give exact estimates for the properties of control zones needed for the eradication of ticks. The numbers obtained in our analysis are based on very rough estimates, and our calculations show that they are quite sensitive to the input data. Further detailed concrete observations can make the estimates more precise and more specific for concrete regions.
On the other hand, we used one-dimensional diffusion models aiming at the approximate description of the propagation in a certain direction. Of course, it would be natural to extend the results to a more realistic (for ticks and many other species) two-dimensional diffusion models. One can also argue that approximations that are more appropriate represent other popular models, like the telegraph equations, or sub-diffusions or super-diffusions.
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