

Behavior changes of epidemic control in condition of sufficient and limited resources

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Motivation and Aim: Epidemic process is the propagation of the pathogen over humans. There are two points of influence on the development of the epidemic process:

1. First, on susceptible individuals by vaccination and pre-contact prophylaxis.
2. Second, on infected individuals by testing, diagnostics, isolation, effective cure and tracing contacts with isolation and diagnostics again.

Influence efficacy on the epidemic process varies. In some circumstances the resources are sufficient, vaccination and isolation are performed with a maximum high rate. In other circumstances the resources are limited. There are limited number of vaccination points of medical workers engaged in vaccination. There can be also limited number of medical and outreach workers with the function to find people with the symptoms of disease. The number of tests can be also limited. If there is a difference in behavior between two systems, up to now it is unknown.

Methods and Algorithms: We compared two variants of maximum deliberately simplified system (MDSS)

MDSS

$$\begin{aligned} X' &= -R\alpha XY + \mu - \mu X - \lambda X^{p_1} \\ Y' &= R\alpha XY - \beta Y - \mu Y - \delta Y^{p_2} \end{aligned}$$

System1 $p_1 = p_2 = 1$

$$\begin{aligned} X' &= -R\alpha XY + \mu - \mu X - \lambda X \\ Y' &= R\alpha XY - \beta Y - \mu Y - \delta Y \end{aligned}$$

System2 $p_1 = p_2 = 0$

$$\begin{aligned} X' &= -R\alpha XY + \mu - \mu X - \lambda \\ Y' &= R\alpha XY - \beta Y - \mu Y - \delta \end{aligned}$$

where X is the proportion of susceptible individuals, Y is the proportion of infected (uncontrolled sources of infection), R is the contact rate, α is the intensity of infectiousness, β is the intensity of recovery, μ is the intensity of the natural movement of the population (birth and death rates, inflow and outflow), δ is the intensity of detection and taking for cure of sources of infection, p_2 is value of δ resources sufficiency, λ is the intensity of vaccination, p_1 is value of λ resources sufficiency. Parameter values as of acute infection: $\alpha = \beta = 0.074$ (1/day), $R = 2$, $\mu = 0.000157$ (1/day), $\lambda = 0.00001$ (individuals/population).

Results: In system 1 (proportionate, sufficient resources) the trace (tr) of the Jacoby matrix (Jac) of the nontrivial equilibrium (NTE) is always below zero, that means that the system is either steady state or unstable with the hard loss of stability while travelling to trivial equilibrium (TE). In figures dash is unstable equilibrium, solid is stable equilibrium.

$$tr(Jac(System1)) = -\frac{\mu R \alpha}{\mu + \beta + \delta}$$

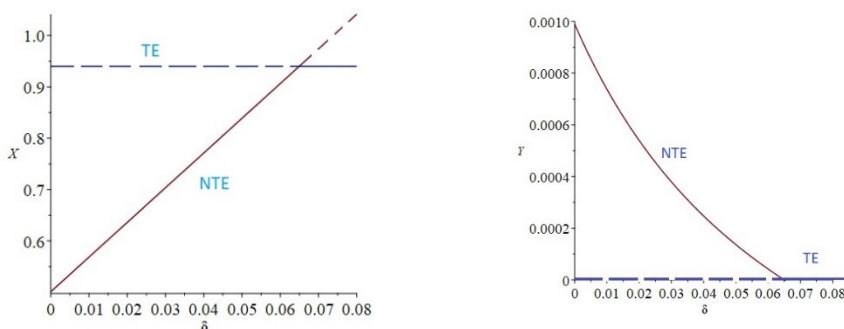


Fig. 1. Bifurcation diagram of system 1 showing control evolution in direction enhancing force δ : X – left, Y – right

In system 2 (unproportionate, insufficient resources) the trace of the Jacoby matrix can be either above or below zero, that completely changes the system behavior. With the growth of δ system passes the unstable cycle (1), mild loss of stability (with oscillations) (2), hard loss of stability (without oscillations) (3).

$$\begin{aligned} tr(Jac(System2)) = & - \left(((-\mu + \delta + \lambda))R^2(\mu - \delta - \lambda)^2\alpha^2 - 2R\mu(\mu + \delta - \lambda)(\beta + \mu)\alpha + \mu^2(\beta + \mu)^2 \right)^{\frac{1}{2}} - \mu^3 \\ & + (R\alpha - \beta - 3\delta + \lambda)\mu^2 + \left(-2R(\delta + \lambda)\alpha - 5\left(\delta - \frac{\lambda}{5}\right)\beta \right)\mu + R(\delta + \lambda)^2\alpha - 2\beta^2\delta \Big) R\alpha / ((\beta \\ & + \mu)(-\mu^2 + (R\alpha - \beta)\mu - R(\delta + \lambda)\alpha \\ & - \left(R^2(\mu - \delta - \lambda)^2\alpha^2 - 2R\mu(\mu + \delta - \lambda)(\beta + \mu + \mu^2(\beta + \mu)^2)^{\frac{1}{2}} \right)) \end{aligned}$$

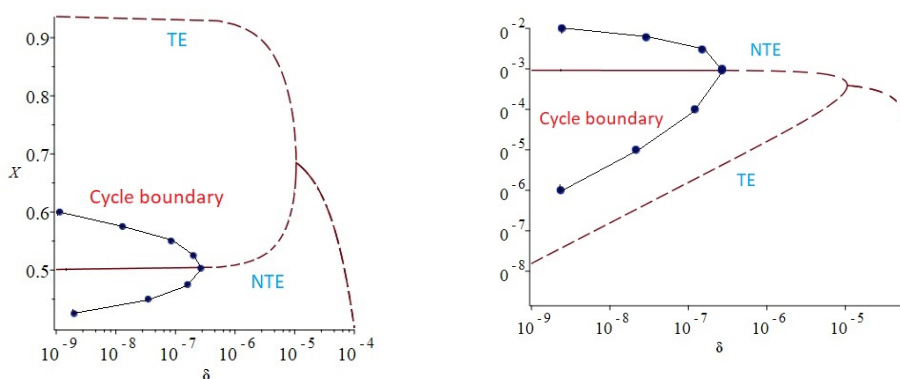


Fig. 2. Bifurcation diagram of unproportionate system 2 showing control evolution in direction enhancing force δ : X – left, Y – right

Control aims satisfies only hard loss of stability.

Conclusion: In sufficient control resources the system is plane and the hard loss of stability occurs after reaching the threshold. In limited resources the system behavior becomes complex and big oscillations may precause bifurcation to the end of the epidemic process.

References

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