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Computation and tracing of periodic solutions of Marchuk–Petrov model

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Motivation and Aim: Models represented by systems of delay differential equations are widely used to study the dynamics of infectious diseases and immune response. Stable periodic and non-trivial stationary solutions to such systems are interpreted as chronic diseases. Therefore, to study chronic diseases, it is necessary to compute the stationary and periodic solutions with sufficiently high accuracy and study their stability. Efficient methods for computing stationary solutions of time-delay systems and analyzing their stability were proposed in [1, 2]. These methods were tested on two well-known models, namely, the model of the dynamics of experimental infection caused by lymphocytic choriomeningitis viruses (LCMV) proposed in [3], and the Marchuk–Petrov model of antiviral immune response (MPM) proposed in [4]. It was shown that the proposed methods made it possible to obtain all stationary solutions of each model for given values of its parameters, trace these solutions along a selected parameter, and study their stability. The report discusses methods for computing periodic solutions of delay systems, proposed in [5, 6] using the Marchuk-Petrov antiviral immune response model as an example, and presents a new method for tracing a given periodic solutions along a selected parameter.

Methods and Algorithms: A periodic solution is suggested to be searched for in vicinity of a given unstable stationary solution in 3 stages: computation of an approximate periodic solution by the relaxation method, estimation of the least period of the approximate periodic solution by the Fourier series expansion, refining of the approximate periodic solution and the least period by a Newton type method. Tracing of a given periodic solution along a selected parameter is performed by constructing a grid along this parameter and computing the solution at each new node of the grid by the Newton type method starting with the solution computed at previous node.

Results: The proposed technology was applied to compute and trace periodic solutions for the Marchuk-Petrov model with parameter values corresponding to hepatitis B. The dependence of the amplitude and period of the solutions on the virus and immune response parameters of the model was examined in detail. The identified relationships provide relevant information for understating the mechanisms underlying relapsing forms of the infection and spontaneous recovery.

Conclusions: The proposed methods make it possible to obtain a periodic solution of the Marchuk-Petrov model in the vicinity of a given unstable stationary solution and to trace the periodic solution along a selected parameter. Application of the methods to a systematic analysis of the model parameter space for existence of qualitatively and quantitatively different types of chronic infection dynamics enables novel understanding of the hepatitis B infection pathogenesis.

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