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Acoustic tomography: inverse problem of source recovery

Guber A.[1]*, Shishlenin M.[1, 2]
[1] *Novosibirsk State University, Novosibirsk, Russia*
[2] *Institute of Computational Mathematics and Mathematical Geophysics, SB RAS, Novosibirsk, Russia*
* *a.guber@g.nsu.ru*

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Motivation and Aim: Acoustic tomography of the lungs is a method of detecting various pathologies of the respiratory system. This is safer than CT, because during the examination a person is not exposed to radiation. Moreover, acoustic tomographs are much cheaper than classical computed tomographs, which makes it possible to massively use acoustic tomography methods for the diagnosis of, for example, malignant tumors or complications of COVID-19 – pneumonia, which is considered to be the most common cause of death of patients.
The report investigates the inverse problem of determining the source of acoustic waves in the three-dimensional case, the solution of which is necessary for a tomographic reconstruction of an acoustic image of the lungs. Similar problems also often arise in orthopedics when determining the degree of joint wear [1, 2], microseismic monitoring of hydraulic fracturing [3], etc.
Methods and Algorithms: Let us consider a direct problem for the acoustics equation in the domain $\Omega = \{(x, y, z): x \in (0, L_x), y \in \left(0, L_y\right), z \in (0, L_z)\}$:

$$u_{tt} = c^2(x, y, z)\left(\Delta u - (\nabla \rho(x, y, z), \nabla u)\right) + q(x, y, z, t), (x, y, z) \in \Omega, t \in (0, T), \quad (1)$$

$$u|_{t=0} = 0, u_t|_{t=0} = 0, \quad (2)$$

$$u|_{\partial\Omega} = 0 \quad (3)$$

by finding of the function $u(x, y, z, t)$ given $q(x, y, z, t), c(x, y, z), \rho(x, y, z)$.

The inverse problem is to determine the source of vibrations $q(x, y, z, t)$ according to the indications of the acoustic sensors located on the chest of the subject:

$$u(x_m, y_m, z_m, t) = f_m(t), m = \overline{1, M}. \quad (4)$$

Here (x_m, y_m, z_m) is the position of m-th receiver, M is the number of receivers. We assume that the functions $c(x, y, z), \rho(x, y, z)$ are known.

The inverse problem of determining the function $q(x, y, z, t)$ after finite-difference discretization is reduced to solving a large-dimensional SLAE.

$$\boldsymbol{A}q = f. \quad (5)$$

Here $\boldsymbol{A}$ is a $M \cdot (N_t - 1)$ by $(N_x + 1) \cdot \left(N_y + 1\right) \cdot (N_z + 1) \cdot (N_t - 1)$ matrix. The system of equations (5) is strongly underdetermined.

The singular value decomposition method and the r-solution are used to solve this SLAE.

Results: The decrease in the condition number of the matrix $\boldsymbol{A}$ of the discrete operator of the inverse problem is investigated, and the behavior of the degree of sparsity of the matrix of the direct problem with an increase in the density of the grid partition over time is studied. Numerical methods for solving direct and inverse problems using OpenMP, MKL and CUDA technologies are implemented. The results of numerical calculations are given.

References

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