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Mathematical and numerical modelling of the circadian oscillator

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We construct smooth 6-dimensional dynamical system of the kinetic type

$$dx_1/dt = k_1(\Gamma_1(x_2)\cdot\gamma_1(x_3) - x_1);\ dx_j/dt = k_j(\Gamma_j(x_5)\cdot L_j(x_1) - x_j);\ j = 2,3,4;$$

$$dx_5/dt = k_5(\Gamma_5(x_6) - x_5);\ dx_6/dt = k_6(C\cdot L_6(x_4) - x_6);\ C = \text{const};\ (1)$$

as a model of circadian oscillator, see [1]. Here, all the variables denote concentrations of the components and are non-negative, all parameters and functions are positive. The functions L_i decrease monotonically and describe negative feedbacks, $i = 2,3,4,6$. The functions γ_1 and Γ_s increase monotonically and describe positive feedbacks in this oscillator, $s = 1,\ldots,5$.

We find conditions of uniqueness and non-stability of an equilibrium point of the system [1], and in this case, we show existence of a cycle of this system near this equilibrium point.

For numerical experiments with trajectories of the system (1), and for visualizations of these results, a specialized cloud application is elaborated on the basis of the R language: https://andreyakinshin.shinyapps.io/clock-bmal1/

Limit cycles and their bifurcations were observed in these experiments for appropriate values of parameters of the system (1).

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