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The optimal feedbacks in the mathematical model of chemotherapy for a nonmonotonic therapy function

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Motivation and Aim: We investigate a pharmacokinetic problem for a deterministic nonlinear system with piecewise monotonic dynamics describing the process of chemotherapy of a malignant tumor. We consider the case when the therapy function, which describes the effect of the drug on the cell growth rate, has two maxima.
Methods and Algorithms: The work presents results of numerical calculation of the optimal result (the value function) and optimal positional strategy of therapy (optimal feedbacks) in a corresponding optimal control problem. The construction use the fact that the value function is the unique minimax (viscosity) solution [1, 2] of the Cauchy problem for the basic Hamilton–Jacobi–Bellman (HJB) equation. By means of the continuous gluing of a finite number of smooth functions obtained by the Cauchy method of characteristics for auxiliary linear HJB equations, the continuous function ϕ is constructed. The paper [3] proves that the constructed function ϕ coincides with the value function.
Results: A new element of the construction is the construction of a line of nonsmooth gluing using the Rankin-Hugoniot conditions [4, 5]. This line plays a key role for the optimal feedback strategy, because it determines its discontinuity line. The results of numerical calculations of the Rankin-Hugoniot line are exposed. Comparison with the results for the case of a single maximum in the therapy function in this model [6] is given.
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