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Asymptotic stability of solutions in one model of disease

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Motivation and Aim: We consider a system of delay differential equations describing the spread of a disease [1]:

$$\frac{d}{dt}x(t) - \sigma - \mu_1 x(t) - \beta x(t)z(t),$$

$$\frac{d}{dt}y(t) = \beta x(t-\tau)z(t-\tau)e^{-\alpha\tau} - \mu_2 y(t),$$

$$\frac{d}{dt}z(t) = \rho y(t) - \mu_3 z(t),$$

where x(t) is the concentration of uninfected cells, y(t) is the concentration of infected cells that produce virus, and z(t) is the concentration of plasma virus. All the parameters of the system are constant and positive. We study the asymptotic stability of stationary solutions to this system.

Methods and Algorithms: When studying asymptotic properties of solutions to systems of nonlinear delay differential equations, in [2] it was proposed a modified Lyapunov–Krasovskii functional. It is important to note that the construction of such functional can be reduced to solving well-conditioned problems and does not require finding roots of quasi-polynomials. We use an analogue of such functional.

Results: We obtain estimates of solutions characterizing the stabilization rate at infinity and establish estimates of attraction domains of asymptotically stable stationary solutions.

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