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The uniqueness of the solution of the two-dimensional direct problem is the propagation of the action potential along the nerve fiber

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Formulation of the problem: The two-dimensional problem of the process of propagation of the action potential along nerve fibers is modeled by the following problem of an equation of parabolic type [1]:

$$C_m(x, y)v_t'(x, y, t) = \frac{r_a(x, y)}{2\rho_a(x, y)}\Delta v - \frac{v(x, y, t)}{\rho_m(x, y)\cdot l}, \ (x, t)\in R_+^2, y\in R, \quad (1)$$

$$v(x, y, t)|t < 0 \equiv 0, \quad v_x'(x, y, t)|x = 0 == h(y)\theta(t) + r(y)\theta_1(t) + p(y)\theta_2(t), \ t\in R_+, y\in R, \quad (2)$$

where, $h(y)$, $r(y)$, $p(y)$ – are given functions, $\theta(t)$ – is the Heaviside theta function, $\theta_1(t) = t\theta(t)$, $\theta_2(t) = \frac{t^2}{2}\theta(t)$, C_m – is the capacitance, r_a – is the radius of the nerve fiber, ρ_a, ρ_m – are the resistivities of the nerve fiber plasma and the membrane material, respectively, l – is the membrane thickness, $v(x, y, t)$ – is the intracellular action potential, a and m – indexes of nerve fibers and membranes, Δv – is the Laplace operator.

Using the Laplace transform, the parabolic problem (1)–(2) is reduced to an equivalent problem of hyperbolic type [2].

For this hyperbolic problem, a theorem on the uniqueness of the solution is justified [3]. It is shown that the uniqueness of the parabolic problem is established from the equivalence of problems of hyperbolic and parabolic type.

References

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