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The optimal control of stochastic differential equations arising in biology, economy and finance

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Motivation and Aim: Real biological, social and economic processes are exposed to external influences that can't always be clearly described (for example, hormonal fluctuations, variations in blood pressure, respiration in biological problems or political changes, the "human factor", natural disasters in social and economic problems). In this connection, there is a growing need to expand deterministic models to models that are described by stochastic differential equations (SDE), where the relevant parameters are modeled as suitable random processes, or stochastic processes are added to the equations of the motion system. The problems of controlling stochastic dynamical systems are widely encountered in practice and are the subject of deep mathematical research. Inverse problem [1] for SDE that consists in determination of the right-hand side function (control function) for the Merton equation with the Wiener process [2] is numerically investigated.
Methods and Algorithms: A numerical regularization algorithm to ensure optimal control is developed. To determine the control function in SDE, various methods are used. The first is to reduce SDE to partial differential equations (the Fokker-Planck equation) [3]; the second is based on the method of dynamic programming. In this paper, the numerical algorithm relies on the principle of dynamic programming (applied to the Merton problem).
Results: A numerical algorithm for solving the inverse problem on the basis of dynamic programming, which determines the optimal control function, is developed.
Conclusion: The developed numerical algorithm allows one to determine the optimal function on the right-hand side in SDE describing real biological, social and economic processes or interactions.
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