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On cycles in models of asymmetric circular gene networks

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Motivation and Aim: We consider nonlinear dynamical systems as models of functioning of asymmetric circular gene networks more complicated and general than analogous models studied in [1–3]. Our main aim here is to find conditions of existence of oscillating trajectories (cycles) of these systems.
Methods and Algorithms: Our constructions and studies of circular gene networks models and description of geometric and combinatorial structures of their phase portraits are based on our previous results, see [3]. In our numerical experiments we used the soft STEP elaborated in the Sobolev institute of mathematics.
Results: For positive parameters k_j and μ_s and positive monotonically decreasing smooth functions f_m, $m = 1, 5, 8$, which describes negative feedbacks in the gene network, we consider 9D-dynamical system

$$\frac{dx_1}{dt} = f_1(x_9) - k_1x_1; \ \frac{dx_j}{dt} = f_j(x_{j-1}) - k_jx_j; j = 5, 8;$$
$$\frac{dx_s}{dt} = \mu_s(x_{s-1}) - k_sx_s; \ s = 2, 3, 4, 6, 7, 9. \qquad (1)$$

Here x_1, x_5, x_8 are concentrations of mRNA's, and all the other variables denote concentrations of proteins which are "intermediate" stages of this gene network functioning. Here, in contrast with [1–3], several intermediate stages can appear between each pair of mRNA's with consecutive indices, not just one.
We show uniqueness of equilibrium point S_0 of the system (1) and find conditions of existence of a cycle $\boldsymbol{C}$ of this system, and describe an invariant polyhedral domain W of this system in the positive octant of 9-D space and contains $\boldsymbol{C}$. These conditions are formulated in terms of matrix of linearization of the system (1) at the point S_0: the non-diagonal non-zero terms of this matrix should be sufficiently large with respect to the parameters k_j, k_s. The invariant domain W is composed by 18 adjacent parallelepipeds and retracts to $\boldsymbol{C}$. Our numerical experiments illustrate and correspond to the theoretical results. We show non-uniqueness of the cycles in some higher-dimensional dynamical systems of the type (1).
Conclusion: In contrast with [2], where the particular case $m_1 = m_2 = m_1 = 1$ symmetric with respect to cyclic permutations of the variables was studied, the shifts along trajectories of the system (1) are not described by equations with delayed arguments. The cycle $\boldsymbol{C}$ is not symmetric with respect to this permutation.
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