DOI 10.18699/MM-HPC-BBB-2018-09

Inverse problems in tomography: an evolutionary approach

V. Dedok
Sobolev Institute of Mathematics SB RAS, Novosibirsk, Russia
e-mail: dedok@math.nsc.ru

Key words: tomography, inverse problems, genetic algorithms

Motivation and Aim: A lot of inverse problems in tomography may be reduced to inverse kinematic problem. In this kind of inverse problem, we assume to know a wave travel time between each pairs of points in the boundary of discovered domain. If a discovered domain with unknown internal structure has a cube form with n^3 elementary cubes we have $O(n^5)$ traces. This large amount of source data makes the problem too hard to solve. Moreover, in practice a wave travel time is unknown, we deal with the phaseless intensity of scattered wave. In this paper we present an effective method of solving of the inverse kinematic problem based on evolutional genetic algorithms.
Methods and Algorithms: Mathematically the inverse problem is formulated the following way. Consider a domain of cube form divided into n^3 elementary cubes with constant refractive index. The problem is to find unknown refractive index in each elementary cube using travel time $\tau_*(x, y)$ between any points on the board of the domain. To get rid of phaseless data we use the method, introduced in [1].
Numerically we need to construct a set of refractive indexes which corresponds to the minimum of residual functional $E = (\tau(x, y) - \tau_*(x, y))^2$. We use a genetic algorithm to find this minimum.
Genetic operations are:

- crossover – average genetic code between two items;
- mutation – random change of genetic code.

The termination condition is a combination of minimum criteria and limited number of generations.
Results: We test our numerical method on computationally simulated data. Numerical studies of the genetic algorithm show its effectiveness on model cases. For the test cases we use homogeneous medias with some spherical heterogeneities with different refractive indexes. The method demonstrates pretty well reconstruction of unknown media.
Conclusion: We show that the genetic algorithms may be an effective method for inverse problem solving. It shows its effectiveness in discovered tomography problem. Unlike traditional optimization methods the genetic algorithm requires fewer computations than the gradient methods. Also, it allows to use undifferentiable functionals like $|\tau(x, y) - \tau_*(x, y)|$ and find solution in different metrics.
Acknowledgements: The work was supported by the comprehensive program of fundamental scientific researches of the SB RAS II.1, project No. 0314-2018-0009, by the RFBR (17-01-00120).

References

1. Romanov V.G. (2017) The problem of recovering the permittivity coefficient from the modulus of the scattered electromagnetic field. Siberian Mathematical Journal. 58(4):711-717.